The Maths Society

Logarithmic and Exponential Functions

Logarithmic and Exponential Functions

1. Solve the equation $log_5(8x + 7) - log_5 2x = 2$.

[3]

$$\log_5(8x+7) - \log_5 2x = 2$$

$$\log_b a - \log_b c = \log_b\left(\frac{a}{c}\right)$$

$$\Rightarrow \log_5\left(\frac{8x+7}{2x}\right) = 2$$

$$\left(\frac{8x+7}{2x}\right) = 5^2$$

$$\frac{8x+7}{2x} = 25$$

$$8x+7 = 50x$$

$$42x = 7$$

$$x = \frac{1}{6}$$

2. (a) Given the simultaneous equations

$$lgx + 2lgy = 1,$$
$$x - 3y^2 = 13,$$

(i) Show that $x^2 - 13x - 30 = 0$.

[4]

$\log x + 2\log y = 1$

$\log(xy^2) = 1$

$xy^2 = 10 \Rightarrow y^2 = \frac{10}{x}$

$x - 3\left(\frac{10}{x}\right) = 13$

$x^2 - 30 = 13x \Rightarrow x^2 - 13x - 30 = 0$

$(c\log_b a = \log_b(a^c))$

$(\log_b a + \log_b c = \log_b(ac))$

(ii) Solve these simultaneous equations, giving your answers in exact form.

[2]

$x^2 - 13x - 30 = 0$

$(x-15)(x+2) = 0$

$x = -2, 15$
(reject)

$x > 0$ as $\log x$ has to be defined

when $x = 15$, $y^2 = \frac{10}{15}$

$= \frac{2}{3} \Rightarrow y = \pm\sqrt{\frac{2}{3}}$ $y > 0$

$\therefore x = 15, y = \frac{\sqrt{6}}{3}$

(b) Solve the equation $log_a x + 3log_x a = 4$, when a is a positive constant , giving x in terms of a.

[5]

$$\log_b a = \frac{1}{\log_a b}$$

$$\log_a x + 3\log_x a = 4$$

$$\Rightarrow \log_a x + 3\left(\frac{1}{\log_x a}\right) = 4$$

Let $y = \log_a x$

$$y^2 + 3 = 4y$$

$$y^2 - 4y + 3 = 0$$

$$(y-1)(y-3) = 0$$

$$\Rightarrow \log_a x = 1, 3$$

$$\therefore x = a, a^3$$

3. (a) Find the exact solution of the equation $2e^{6x} - 3e^{3x} - 5 = 0$.

[3]

Let $y = e^{3x}$

$2y^2 - 3y - 5 = 0$

$(2y-5)(y+1) = 0$

$y = -1, \frac{5}{2}$ $\quad y > 0$ as $e^{3x} > 0$

$e^{3x} = \frac{5}{2} \Rightarrow x = \frac{1}{3}\ln\frac{5}{2}$

(b) Solve the following simultaneous equations.

$$e^{4x-7} \div e^{5x+7y} = \frac{1}{e^2}$$

$$xy + 18 = 0$$

[5]

$e^{(4x-7)-(5x+7y)} = e^{-2}$

$4x - 7 - 5x - 7y = -2$

$x = -5 - 7y$

$y(-5-7y) + 18 = 0$

$-5y - 7y^2 + 18 = 0$

$(7y-9)(y+2) = 0$

$y = -2, \frac{9}{7}$

when $y = -2$, $x = -5 - 7(-2)$
$= 9$

when $y = \frac{9}{7}$, $x = -5 - 7(\frac{9}{7})$
$= -14$

4. (a) Solve the equation $5^{w-1} = 12$, giving your answer correct to 2 decimal places. [2]

$w-1 = \log_5 12$

$w = 1 + \log_5 12 = 2.54$ (2dp)

(b) Solve the equation $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 6 = 0$. [3]

let $y = x^{\frac{1}{3}}$

$y^2 - 5y + 6 = 0$

$(y-3)(y-2) = 0$

$y = 2, 3$ $\Rightarrow x = 8, 27$

5. (a) Write $2lgx - (lg(x+6) + lg3)$ as a single logarithm to base 10. [2]

$2\log x - (\log(x+6) + \log 3)$

$\Rightarrow \log x^2 - \log[3(x+6)] = \log\left[\frac{x^2}{3(x+6)}\right]$

(b) Hence solve the equation $2lgx - (lg(x+6) + lg3) = 0$ [4]

$\log\left[\frac{x^2}{3(x+6)}\right] = 0$

$\frac{x^2}{3(x+6)} = 1$

$x^2 = 3x + 18$

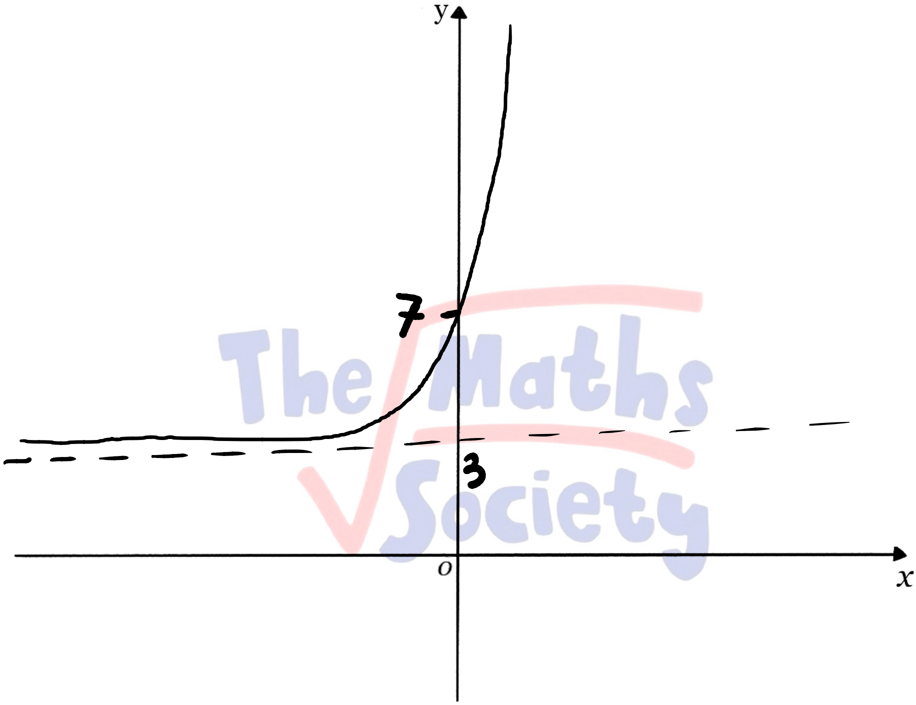
$x^2 - 3x - 18 = 0$

$(x-6)(x+3) = 0$

$x = -3, 6$ but $x > 0$ $\therefore x = 6$

(c) On the axes, sketch the graph of $y = 4e^{x} + 3$ showing the values of any intercepts with the coordinate axes.

[2]



6. (a) (i) Write down the set of values of x for which $lg(5x - 3)$ exists.

[1]

$5x - 3 > 0$

$x > \frac{3}{5}$

(ii) Solve the equation $lg(5x - 3) = 1$.

[1]

$\log(5x - 3) = 1$

$5x - 3 = 10$

$5x = 13$

$x = 13/5$

(b) It is given that $log_y x = 4 + \frac{1}{2}log_y 64 + log_y 162$, where $y > 0$. Find an expression for y in terms of x. Simplify your answer.

[5].

$\log_y x = 4 + \frac{1}{2}\log_y 64 + \log_y 162$

$= 4 + \log_y 8 + \log_y 162$

$\log_y x = 4 + \log_y 1296$

$\log_y x = 4\log_y y + \log_y 1296$

$\log_y x = \log_y y^4 + \log_y 1296$

$\log_y x = \log_y 1296y^4$

$x = 1296y^4$

$y^4 = \frac{x}{1296}$

$y = \frac{x^{1/4}}{6}$